Exercise 28

Find $f'(x)$ and $f''(x)$.

$$f(x) = \sqrt{x}e^x$$

Solution

Use the product rule to differentiate $f(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(x^{1/2}e^x\right) \\ &= \left[\frac{d}{dx}(x^{1/2})\right](e^x) + (x^{1/2})\left[\frac{d}{dx}(e^x)\right] \\ &= \left(\frac{1}{2}x^{-1/2}\right)(e^x) + (x^{1/2})(e^x) \\ &= \left(\frac{1}{2}x^{-1/2} + x^{1/2}\right)e^x \end{aligned}$$

Use the product rule again to differentiate $f'(x)$.

$$\begin{aligned} f''(x) &= \frac{d}{dx}\left[\left(\frac{1}{2}x^{-1/2} + x^{1/2}\right)e^x\right] \\ &= \left[\frac{d}{dx}\left(\frac{1}{2}x^{-1/2} + x^{1/2}\right)\right](e^x) + \left(\frac{1}{2}x^{-1/2} + x^{1/2}\right)\left[\frac{d}{dx}(e^x)\right] \\ &= \left(-\frac{1}{4}x^{-3/2} + \frac{1}{2}x^{-1/2}\right)e^x + \left(\frac{1}{2}x^{-1/2} + x^{1/2}\right)(e^x) \\ &= \left(-\frac{1}{4}x^{-3/2} + x^{-1/2} + x^{1/2}\right)e^x \end{aligned}$$